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Precalculus | Packer Collegiate Institute

## Assessment on Rational Functions

Remember to get full credit, you need to show all work, clearly and neatly. Remember, this isn't just about you getting the answer, but you showing someone else how you got the answer.

You may not use a calculator on this assessment.

1. (a) Draw two functions $f(x)$ and $g(x)$ so that their quotient gives us the graph that results.

(b) Explain why you have that hole in the quotient graph, and why it appears where it does, based on the previous two graphs. Use the graphs to explain your answer, not algebraic equations.
2. Write an equation for the following two rational functions. You may leave the numerator and denominator in factored form.


Equation:


Equation:
3. Explain as well as you can why $f(x)=\frac{1}{x}$ has a vertical asymptote at $x=0$. In your response, be sure to explain what a vertical asymptote is graphically, and how the equation gives rise to that sort of graphical behavior.
4. (a) Write an equation of a rational function with vertical asymptotes at $x=-2$ and $x=3$, a hole at $x=5$, and x-intercepts at $x=-6$ and $x=7$.
(b) What the domain of that function you created in part (a)? Give your answer in interval notation.
5. Explain as clearly as you can, to someone in Algebra II, why $f(x)=\frac{x^{2}-5}{x^{2}+10}$ will have a horizontal asymptote at $y=1$.
6. Find the end behavior of the following (if there is a horizontal asymptote, list it, if there is no horizontal end behavior, explain what's going on for very large positive and very large negative $x$-values.

| $f(x)=\frac{3 x^{3}-x-1}{5 x^{2}+x^{5}}$ | $g(x)=\frac{4 x-1}{5 x+1}$ | $h(x)=\frac{2 x-3}{3 x^{2}+x-2}$ |
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7. The purpose of sign analyses are to give us a sense of when a rational function changes from being above the $x$ axis to below the $x$-axis. They can tell us quite a lot about the shape of a graph, even if we don't know anything else. We determined there were only three features on a graph that might result in a sign change.

These features are: $\underline{x}$-intercepts, $\qquad$ and $\qquad$ .

Pick one of those other two features and draw an example of a graph where there is a sign change at that feature, and an example of a graph where there is not a sign change at that feature.

The feature I chose is: $\qquad$

| $\ldots$ is a sign change |
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|  |

... is not a sign change
8. For the following rational functions, find the x-intercepts, the holes, the vertical asymptotes, the sign analysis, and the end behavior. Then come up with a decent sketch of the graph.
(a) $f(x)=\frac{x^{2}-1}{x^{2}-3 x-4}$

(b) $g(x)=\frac{x+2}{x^{2}-9}$

9. Let $h(x)=\frac{2(x+1)(x-8)}{(x-2)^{2}}$.

## WORK SPACE

(a) When is $h(x) \geq 0$ ?
(b) When is $h(x)<0$ ?
10. (a) When graphed, the rational function $r(x)=\frac{(x+1)(x-4)}{(x+1)}$ will look like:
(b) The rational function $r(x)$ has a hole. What is the height of the hole? How do you know?

Extra Credit: Find a reasonable equation for:


## INTEGRITY STATEMENT:

"On my personal integrity, I have not given, nor received, nor witnessed any unauthorized assistance on this exam."
(signature)
If you can't sign this in good conscience, please don't. Come speak to me.

