# Multivariable Calculus: Problem Set 1B

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Instructions: You are encouraged to discuss general strategies to approach the questions on this problem set with your classmates, but you must work and write up your solutions to the problems entirely on your own. Of course, you are always welcome to meet with me to talk about any question you are having difficulty with. Please pay attention to making your solutions as clear as possible for the reader; mathematical communication is an important skill that you will develop in this course.

PROBLEM 1

Suppose that $\vec{a}$, $\vec{b}$, and $\vec{c}$ are mutually perpendicular, as shown in the figure below. Set S_F be the area of face F. Prove the following three-dimensional version of the Pythagorean Theorem:

$$S_A^2 + S_B^2 + S_C^2 = S_D^2$$
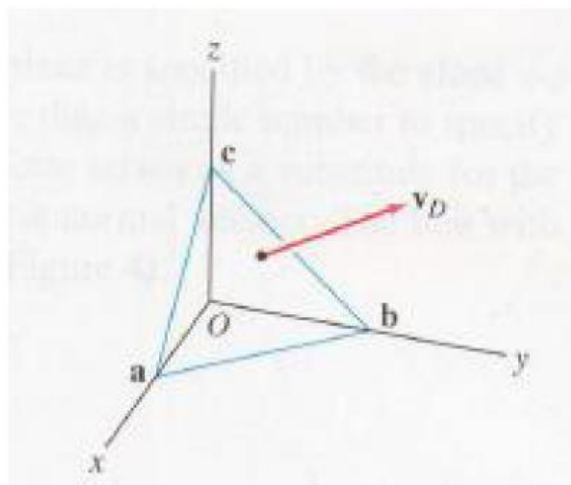
PROBLEM 2

Two bugs are walking along lines in 3-space. At time t bug 1 is at the point (x, y, z) on the line $x = 4 - t, y = 1 - t, z = 2 + t$. At the same time t, bug 2 is at the point (x, y, z) on the line $x = t, y = 1 + t, z = 1 + 2t$.

Assume that the distance is in centimeters and that the time is in minutes.

(a) Find the distance between the bugs at time $t = 0$.

(b) Use a graphing utility to graph the distance between the bugs as a function of time from $t = 0$ to $t = 5$. Please print out a copy of your graph. (You may use an online graphing program, you may use WINPLOT, or you may also use the virtual TI on your computer, and save the image of your plot by right clicking on the graph and choosing "Take Screenshot" and choosing "LCD only, black/white (BMP)".)

(c) What does the graph tell you about the distance between the bugs.

(d) How close to the bugs get? (Please be exact.)

PROBLEM 3

Find the parametric equations of the line of intersection of the planes $-2x+3y+7z+2=0$ and $x+2y-3z+5=0$.

PROBLEM 4

The earth's rotation causes a flattening at the poles, so its shape is often modeled as an oblate spheroid rather than a sphere (see Exercise 54 of 831 of our textbook). One of the models used by global positioning satellites is the *World Geodesic System of 1984 (WGS-84)*, which treats the Earth as an oblate spheroid whose equatorial radius is 6378.1370 km and whose polar radius (the distance from the Earth's center to the poles) is 6355.5231 km.

(a) Use the WGS-84 model to find an equation for the surface of the Earth.
(b) Use your knowledge of single variable calculus to calculate the surface area of the Earth predicted by the WGS-84 model. You will have to use your calculator's *fnInt* function. If you want more information on calculating the surface area of an oblate spheroid, go to the following link:

   http://mathworld.wolfram.com/OblateSpheroid.html

(c) A long winded excerpt from *Foucault's Pendulum*[1]:

> "I imagine your author holds that the height of the pyramid of Cheops is equal to the square root of the sum of the areas of all its sides. The measurements must be made in feet, the foot being closer to the Egyptian and Hebrew cubit, and not in meters, for the meter is an abstract length invented in modern times. The Egyptian cubit comes to 1.728 feet. If we do not know the precise height, we can use the pyramidion, which was the small pyramid set atop the Great Pyramid, to form its tip. It was of gold or some other metal that shone in the sun. Take the height of the pyramidion, multiply it by the height of the whole pyramid, multiply the total by ten to the fifth, and we obtain the circumference of the earth. What's more, if you multiply the perimeter of the base by twenty-four to the third divided by two, you get the earth's radius. Further, the area of the base of the pyramid multiplied by ninety-six times ten to the eighth gives us one hundred and ninety-six million eight hundred and ten thousand square miles, which is the surface area of the earth. Am I right?"

According to Wikipedia[2], the surface area of the Earth is 510,072,000 square kilometers. Assuming Wikipedia has the most accurate information, which estimate of the Earth's surface area is better – your calculation in part (b) or Umberto Eco's in *Foucault's Pendulum*. Explain your answer.

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[1] On page 278, Umberto Eco, *Foucault's Pendulum* (Orlando: Harcourt, Inc., 1989)
[2] http://en.wikipedia.org/wiki/Earth, accessed September 15, 2008

PROBLEM 5

The accompanying figure shows a right circular cylinder of radius 10 cm spinning at 3 revolutions per minute about the z-axis. At time $t = 0$, a bug at the point $(0,10,0)$ begins walking straight up the face of the cylinder at the rate of 0.5 cm/min.

3 rev/min

$(0, 10, 0)$
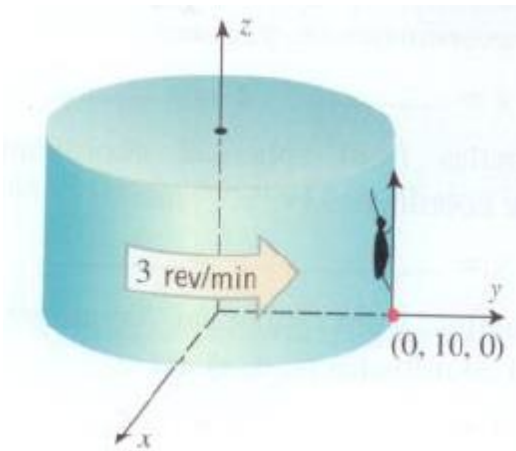
(a) Find the cylindrical, rectangular, *and* spherical coordinates of the bug after 2 minutes.

(b) Use a graphing utility to graph the bug's distance from the origin as a function of time. Please print out and turn in your graph. Be sure to label the scale on the axes. [Again, you may use an online graphing program, or you can use the virtual TI on your computer, and save the image of your plot by right clicking on the graph and choosing "Take Screenshot" and choosing "LCD only, black/white (BMP)".]